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FRACTURE & DYNAMICS

PAPER NO. 26

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Abstract

The Random Dec Technique is a versatile technique for characterization of random signals in the time domain. In this paper a short review of the most important proporties of the technique is given. The review is mainly based on recently achieved results that are still unpublished [15], or that has just been published [14]. In the review theoretical results are given, including results for general trig conditions, fundamental solutions for the case of Gaussian processes, and closed form solutions for the variance of the Random Dec signature. The potential of the technique is illustrated by application of the level trig condition on simulated data. The technique proves to be accurate and fast compared to traditional FFT analysis.

1. Introduction

The Random Dec Technique was developped at NASA in the late sixties and early seventies by Henry Cole and co-workers [1-4]. The purpose was to develop a simple data analysis algorithm for the characterization of stochasic response of space stuctures and aeroelastic systems and to identify damage in such systems by identifying system changes.

The technique has been used for system identification of large structures, Ibrahim [5], for structural damage detection and determination of fluid damping, Yang, [6-8], and for vehicle system identification and damping measurements of soil [9-10].

The basic idea of the technique is to estimate a co-called Random Dec signature which can be used to characterize stochastic time series. If the time series $x(t)$ is given the Random Dec signature $\hat{D}(\tau)$ is formed by averaging N segments of the time series

$$\hat{D}_{XX}(\tau) = \frac{1}{N}\sum_{i=1}^{N} x(\tau - t_i) \qquad (1)$$

where the time series at the times t_i satisfies a certain condition, the so-called trig condition. The condition might be that $x(t_i) = a$ (the level crossing condition), or that $x(t_i) = 0 \wedge \dot{x}(t_i) > 0$ (the zero crossing condition with positive slope) or some similar condition. In eq. (1) an auto signature is estimated since the accumulated average and the condition is applied to the same time series. If for instance the data segments are taken from a time series x_i and the trig condition is applied to a correlated time series y_i the cross signature $\hat{D}_{XY}(\tau)$ is estimed.

The advantage of the technique is that is establishes a basis for simple and fast on-line system identification. Because of the simple algorithm it is possible to make on-line measurements using simple equipment, and because the method works in the time domain it translates the irregular time series into a form that is intuitively meaningful to the observer - a form that is often more usefull when identifying changes in damping ratio than power spectra estimated by the FFT technique.

However, one of the problems of the technique is that a sound theoretical basis has not been fully established yet. In all the references mentioned above, the authors argue on a more or less heuristic basis that the Random Dec signature formed by averaging time series segments from the output of a stochasic loaded system should describe system proporties only. This was shown to be incorrect by Vandiver et al, [11], who proved that under certain conditions (applying the level crossing trig condition to a Gausian process) the Random Dec signature is simply proportional to the correlation function.

In Brincker et al [15] the results of Vandiver are generalized to the case of cross signatures, and general trig conditions. Furthermore in [15] some relations are given for general processes and general trig conditions, and solutions for variance and bias introduced by finite size trig windows are allso derived in [15]. In the following these results are reviewed and the potential of the technique is illustrated by application of the level trig conditions on simulated time series.

2. Theoretical Basis

In the general case the Random Dec (RDD) signature is a function in the time domain characterizing the stationary stochastic processes $X_i(t)$ and $X_j(t)$ defined in the following way

$$D_{X_iX_j}(\tau) = E[X_i(t+\tau)|\ C_{X_j(t)}] \qquad (2)$$

where $C_{X_j(t)}$ is some condition on the process $X_j(t)$ at time t. For intance the condition $C_{X_j(t)}$ might be the level crossing condition $C_{X_j(t)} \Leftrightarrow X_j(t) = a$ or it might be the zero crossing condition with positive slope $C_{X_j(t)} \Leftrightarrow X_j(t) = 0 \wedge \dot{X}_j(t) > 0$.

2.1 The level crossing condition

One of the most important trig conditions is the level-crossing condition for wich the Random Dec signatures are defined by

$$D_{X_iX_j}(\tau,\ a) \;=\; E[X_i(t+\tau)|\ X_j(t) = a] \tag{3}$$

Where the event $X_j(t) = a$ is interpreted as out-crossing through a vertical window. This special condition is important of two reasons. First it is one of the most used conditions in practical applications of the Randon Dec principle, and second, in this case there exsists a simple relation between the correlation functions and the Random Dec signatures, Brincker et al [15]

$$R_{X_iX_j}(\tau) \;=\; \int_{-\infty}^{\infty} s f_{X_j}(s) D_{X_iX_j}(\tau, s)\ ds \tag{4}$$

where $f_{X_j}(.)$ is the marginal distribution of $X_j(t)$. This relation shows that if the shape of the signature is invariant to changes in the trigger level a, then the signatures may be written as a product of a shape function $d_{X_iX_j}(\tau)$ and a function of the trigger level $e_{X_iX_j}(a)$

$$D_{X_iX_j}(\tau, a) \;=\; d_{X_iX_j}(\tau) e_{X_iX_j}(a) \tag{5}$$

and eq. (4) yields

$$R_{X_iX_j}(\tau) \;=\; d_{X_iX_j}(\tau) \int_{-\infty}^{\infty} s f_{X_j}(s) e_{X_iX_j}(s)\ ds \tag{6}$$

which means that the Random Dec signature is simply proportional to the correlation function. In practical application of the Random Dec Technique it should allways be investigated to what degree the shape invarianse of the signature is preserved. If the shape invarianse is not preserved, i.e. eq. (5) is not satisfied, then the technique should be used with care.

It can be shown, Brincker et al [15] that if $X(t)$ is a system input and $Y(t)$ is a system output and if the input signatures D_{XX} and D_{XY} are shape invariant, then the corresponding output signatures D_{YX} and D_{YY} will also be shape invariant. Furthermore if the processes $X_1(t)$ and $X_2(t)$ are uncorrelated and each satisfy the shape invariance condition, then it can be shown that the product $U(t) = X_1(t)X_2(t)$ will also satisfy the shape invariance condition.

2.2 Gaussian processes

Vandiver et al, [11], but also Papoulis, ([13], paragraph 11-3) have shown that if $X(t)$ is a stationary zero mean Gaussian process with variance σ^2 and auto correlation function $R_{XX}(\tau)$, then the auto signature $D_{XX}(\tau)$ for the level trigging condition is given by

$$D_{XX}(\tau,\ a) \ = \ \frac{R_{XX}(\tau)}{\sigma^2}\ a \tag{7}$$

In these derivations the joints normal density function was used directly to arrive at eq. (7). The result is not difficult to generalize to the case of correlated prosseses using the joint normal density function.

However, a different approach can be followed that does not involve the normal density function, but only the proporties of Gaussian prosseses. Furthermore the results of this approach, which are presented in the following, are not restricted to the level crossing condition, but can be generalized by the use of the total representation theorem, se e.g. Ditlevsen [16] p 56.

Let us consider two stationary zero mean Gaussian prosesses $X_i(t)$ and $X_j(t)$. Then by forming the covariance matrix

$$\underline{\underline{C}} \ = \ Cov[\begin{bmatrix} X_j(t_1) \\ X_i(t_2) \\ X_j(0) \\ \dot{X}_j(0) \end{bmatrix} [X_j(t_1)\ X_i(t_2)\ X_j(0)\ \dot{X}_j(0)]] \tag{8}$$

and using the results given in appendix A, the signature (the conditional mean) $D_{X_iX_j}(\tau)$ is found to

$$\begin{aligned} D_{X_iX_j}(\tau) \ &= \ E[X_i(t+\tau)|\ X_j(t) = a \wedge \dot{X}_j(t) = v] \\ &= \ \frac{R_{X_iX_j}(\tau)}{\sigma^2_{X_j}} a - \frac{R'_{X_iX_j}(\tau)}{\sigma^2_{\dot{X}_j}} v \end{aligned} \tag{9}$$

and the variance of the estimate (conditional varianse for the sum of N identical prosesses)

$$\hat{D}_{X_iX_j}(\tau) \ = \ \frac{1}{N}\sum_{k=1}^{N} X_i^k(t+\tau)|\ X_j(t) = a \wedge \dot{X}_j(t) = v \tag{10}$$

is found to

$$Var[\hat{D}_{X_iX_j}(\tau)] = \frac{\sigma_{X_i}^2}{N}(1-(\frac{R_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{X_j}})^2-(\frac{R'_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{\dot{X}_j}})^2) \quad (11)$$

where $R'_{X_iX_j}(\tau)$ is the derivative of the correlation function. The equations (9) and (11) are a kind fundamental solutions for the Random Dec Technique. They are not used in that form in practical applications, since in practical applications one would not condition on both $X(t)$ and $\dot{X}(t)$, but use a trig condition allowing for a suitable number of triggings in the time series. However, from the fundamental solutions (9) and (11) closed form solutions for the Random Dec Signature and the variance on the signature can easily be derived for the most common trig conditions. Two cases will be considered here, the level trigging condition allready considered in the previous section, and another popular condition, the condition of zero crossing with positive slope.

If the level crossing condition is considered, that is $C_{X(t)} \Leftrightarrow X(t) = a$, then we have no condition on $\dot{X}(t)$ and since the processes are Gaussian $\dot{X}_j(t)$ is indepentend of $X_j(t)$

$$\begin{aligned} D_{X_iX_j}(\tau) &= E[X_i(t+\tau)|\ X_j(t)=a] \\ &= \frac{R_{X_iX_j}(\tau)}{\sigma_{X_j}^2}a - \frac{R'_{X_iX_j}(\tau)}{\sigma_{\dot{X}_j}^2}E[\dot{X}_j(t)|\ X_j(t)=a] \\ &= \frac{R_{X_iX_j}(\tau)}{\sigma_{X_j}^2}a \end{aligned} \quad (12)$$

which is the generalized form of eq. (7). The variance on the signature is found as the variance given by eq. (11) plus an additional term caused by the randomization of $\dot{X}_j(t)$

$$\begin{aligned} Var[\hat{D}_{X_iX_j}(\tau)] &= \frac{\sigma_{X_i}^2}{N}(1-(\frac{R_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{X_j}})^2-(\frac{R'_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{\dot{X}_j}})^2) + \\ &\quad + \frac{1}{N}Var[\frac{R'_{X_iX_j}(\tau)}{\sigma_{\dot{X}_j}^2}\dot{X}_j(t)|\ X_j(t)=a] \\ &= \frac{\sigma_{X_i}^2}{N}(1-(\frac{R_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{X_j}})^2) \end{aligned} \quad (13)$$

Note that the variance is independend of the trig level, and that for auto signatures the variance is zero for $\tau = 0$. However, this is expected since all the averaged data segments are forced to have the same initial value.

If the zero crossing condition with positive slope is considered, that is $C_{X(t)} \Leftrightarrow X(t) = 0 \wedge \dot{X}(t) > 0$, then we have the similar results

$$
\begin{aligned}
D_{X_iX_j}(\tau) &= E[X_i(t+\tau)|\ X_j(t)=0 \wedge \dot{X}_j(t)>0] \\
&= -\frac{R'_{X_iX_j}(\tau)}{\sigma^2_{\dot{X}_j}}\ E[\dot{X}_j(t)|\ X_j(t)=0 \wedge \dot{X}_j(t)>0] \\
&= -\sqrt{\frac{2}{\pi}}\,\frac{R'_{X_iX_j}(\tau)}{\sigma^2_{\dot{X}_j}}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
Var[\hat{D}_{X_iX_j}(\tau)] &= \frac{\sigma^2_{X_i}}{N}\,(1-(\frac{R_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{X_j}})^2-(\frac{R'_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{\dot{X}_j}})^2)+ \\
&\quad +\frac{1}{N}Var[\frac{R'_{X_iX_j}(\tau)}{\sigma^2_{\dot{X}_j}}\dot{X}_j(t)|\ X_j(t)=0 \wedge \dot{X}_j(t)>0] \\
&= \frac{\sigma^2_{X_i}}{N}\,(1-(\frac{R_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{X_j}})^2-\frac{2}{\pi}(\frac{R'_{X_iX_j}(\tau)}{\sigma_{X_i}\sigma_{\dot{X}_j}})^2)
\end{aligned} \tag{15}
$$

As a main result we can conclude that for stationary Gaussian prosesses the Random Dec signature will allways be a linear combination of the corresponding correlation function and its first derivative. The level crossing condition picks out the correlation function itself and the zero crossing condition with positive slope picks out the derivative of the correlation function.

3. Implementation problems

Even though the basic idea and the algorithm of the Random Dec Technique is very simple, there is some problems in implementing the technique. Only a very short review will be given here, but the problems of implementation are discussed in detail in Brincker et al [15].

The basic problem is how to implement the trig conditions without introducing to much variance and more important: without introducing bias. An additional problem is the selection of trig points. Some times it is not possible to use all the trig points in a time series, and excluding some of the trig points might bias the Random Dec estimate of the correlation functions. These problems are illustrated in Brincker et al [14] where some numerical results are given, and in the next section where the numerical results are reviwed.

First of all the condition $C_{X(t)} \Leftrightarrow X(t)=a$ is not well defined. A precise definition of the event $X(t)=a$ is given by outcrossing through a window in the limit where the size of the window approaches zero. However the window might be vertical, horizontal or even a general slanted window, and the meaning of the condition $C_{X(t)} \Leftrightarrow X(t)=a$ will depend upon the slope of the window.

However, in practical applications, it is not possible to use any of the definitions given above, since finite size windows must be applied.

The simplest case of a finite size window is the vertical window with height δa introduced by the condition $X(t) \in I$ where $I = [a\ ; a + \delta a]$. In this case we only have that $P[X(t) \in I] \cong f_X(a)\delta a$, and the earlier results for the interpretation af the signature, eq. (12) and the variance on the signature, eq. (13) must be modified.

However, it is not difficult to show that for the finite height window the signature can be interpreted just as given by eq. (12) if only the trig level a is substituted by the expectation value $\bar{a} = E[X_j(t)|\ X_j(t) \in I]$ and that the varianse on the estimate is given by eq. (13) plus an additional term σ_W introduced by the window given by

$$\sigma_W^2 = \frac{1}{N}\,\bar{\sigma}_j^2\,(\frac{R_{X_iX_j}(\tau)}{\sigma_{X_j}^2})^2 \tag{16}$$

where $\bar{\sigma}_j^2 = Var[X_j(t)|\ X_j(t) \in I]$. If the processes are Gaussian, the conditional distribution $f_{X|X\in I}(x)$ is a truncated normal distribution. If the window height δa is small compared to the standard deviation σ_{X_j}, then the conditional distribution will be approximately uniform, and therefore the quantities $\bar{a}$ and $\bar{\sigma}_j$ may be approximated by

$$\bar{a} = a + \frac{\delta a}{2}\ ;\ \ \bar{\sigma}_j^2 = \frac{\delta a^2}{12} \tag{17}$$

Using a horizontal window the level crossing condition (assuming positive slope) $C_{X(t)}^{H+} \Leftrightarrow X(t) = a \wedge \dot{X}(t) > 0$ is defined by $C_{X(t)}^{H+} \Leftrightarrow \lim_{\delta t \to 0}(X(t) < a) \wedge (X(t + \delta t) > a)$, and for Gaussian processes results similar to those for the vertical window now are

$$\bar{a} = a\ ;\ \ \bar{\sigma}_j^2 = \frac{\delta t^2}{3}\,\sigma_{\dot{X}}^2 \tag{18}$$

Results for the general slanted window can also be given, Brincker et al [15].

However, the most important from an application point of view is to avoid bias. It turns out that the bias problems are connected with the loss of information due to the sampling. The problem is illustrated by the following simple example.

It is assumed that Random Dec Signatures are estimated on stationary, zero mean Gaussian processes, and that the continous processes $X(t), Y(t)$ are mapped on the discrete time series $x_i, y_i \in \{a_j\};\ a_j - a_{j-1} = \Delta a$ by analog to digital convertion, where Δa is the resolution of the analog to digital converter and where the time spacing Δt between sample points is determined by the user.

Let us consider the case of detecting level crossings by a horizontal window. The triggings are detected on a time series y_i that is a realization of the process $Y(t)$ and the data segments are taken from a time series x_i that is a realization of the process $X(t)$ and the up-crossings and the down-crossings are detected by separate windows $C_{y_i}^{H+}$ and $C_{y_i}^{H-}$

$$\begin{aligned} C_{y_i}^{H+} &\Leftrightarrow y_i < a \ \wedge \ y_{i+w} > a \\ C_{y_i}^{H-} &\Leftrightarrow y_i > a \ \wedge \ y_{i+w} < a \ ; \ a \in \{a_j\} \end{aligned} \tag{19}$$

where $w\Delta t$ is the width of the window. The resultant window is $C_{y_i}^{H} \Leftrightarrow C_{y_i}^{H+} \vee C_{y_i}^{H-}$ and the Random Dec signature is estimated by

$$\hat{D}_{XY}(m\Delta t) = \frac{1}{N}\sum_{i=1}^{N} S_{w\Delta t/2}\{x_{m-i}\}|\ C_{y_i}^{H}; \ -M \le m \le M \tag{20}$$

where $S\{.\}$ is the shift operator defined by $S_\tau\{f(t)\} = f(t-\tau)$ and N is the number of trig points.

Note that by eq. (20) the trig point (corresponding to the points t_i in eq. (1)) is placed in the midle of the window.

In the case of simultanous sampling the total symmetry of the problem ensures that the statistical proporties of the up-crossing and the down-crossings are identical, and thus the estimate given by eq. (20) will be unbiased, i.e. the expectation of the estimator will be given by eq. (12).

However, in an application one would try to minimize the variance introduced by the window, se eq. (18), and therefore it is tempting to use a horizontal window with the minimum possible width Δt and then move the trig point to the end of the window. In this case symmetriy is lost (the distribution of $x_i|\ C_{x_i}^{H}$ will be different for the up-crossings and the down-crossing), and therefore the estimate becomes biased. Moving the trig point to the end of the window simply shifts the signature, and thus

$$\begin{aligned} D_{XY}(m\Delta t) &= \frac{R_{XY}(m\Delta t + \Delta t/2)}{\sigma_Y^2} a \\ &\cong \frac{R_{XY}(m\Delta t)}{\sigma_Y^2} a + \frac{R'_{XY}(m\Delta t)}{\sigma_Y^2}\frac{\Delta t}{2} \end{aligned} \tag{21}$$

Some numerical results for this case is given in Brincker et al, [14] and in the next section . As expected from the results in section 2.2, se eq. (9), the bias term is proportional to the derivative of the correlation function.

4. Numerical results

In the following some numerical results are given for the estimation of auto correlation functions for the response of a single degree of freedom system loaded by Gaussian white noise. The investigated is reported in Brincker et al [14]. By simulating the response X(t) of the system using a ARMA (2,1) model the exact autocorrelation function $R_{XX}(\tau)$ and estimates $\hat{R}_{XX}(\tau)$ of the autocorrelation function could be compared directly and the mean square error could be calculated without influence from the sampling.

The acuracy and speed of the RDD technique was compared to traditional FFT analysis. The cyclic egenfrequency of the system was $\omega_0 = 12.57$ rad/s, the time spacing Δt between sample points was 0.051 seconds, the length of the time series was 4000 points and the damping ratio was varied trough the values 1 %, 5 % and 25 %. The FFT estimates were obtained by averaging FFT power spectral estimates and finally transform to the time domain by inverse FFT. The RDD signatures where estimated as prescribed by eq. (1) using the level trig condition implemented by a horisontal window and restricting the time paramater to the discrete values $\tau = m\Delta t;\ -M \leq m \leq M$. Some typical results are shown in figure 1 and the speed and accuracy of the two technologies are illustrated in figure 2.

From these results it is seen that the errors on the RDD estimates are generally smaller than the errors on the FFT estimates. Especially in the case were M is small (short estimates) the RDD estimates are substantially better than the FFT estimates. The accuracy of the FFT might be improved by introduction of a spectral window. However, that will be at the expense of the speed.

The speed of the RDD technique is also high compared to the FFT technique; again the great advantage of the RDD technique is when M is small - for the FFT technique the CPU-time increases as M approches zero, whereas for the RDD technique the CPU-time is approximately proportional to M. The RDD algorithm was implemented as a floating point implementation. However, because of the simplicity of the RDD algorithm it is possible to implement the RDD algorithm as an integer implementation making the algorithm much faster. Therefore, the absolute speed results only show that it is not difficult to make the RDD algorithm faster than an FFT algorithm.

The problems of implementing the technique without introducing bias as mentioned in the preceeding section are illustrated in figure 3 and 4.

Figure 3 illustrates bias introduced by the trig window. As mentioned in section 3, when signals are sampled, a finite length window must be applied. A horizontal window was used as given by eq. (20) with the minimum lenght $w = 1$. If the trig point is placed at one of the end points of the trig window, bias is introduced as

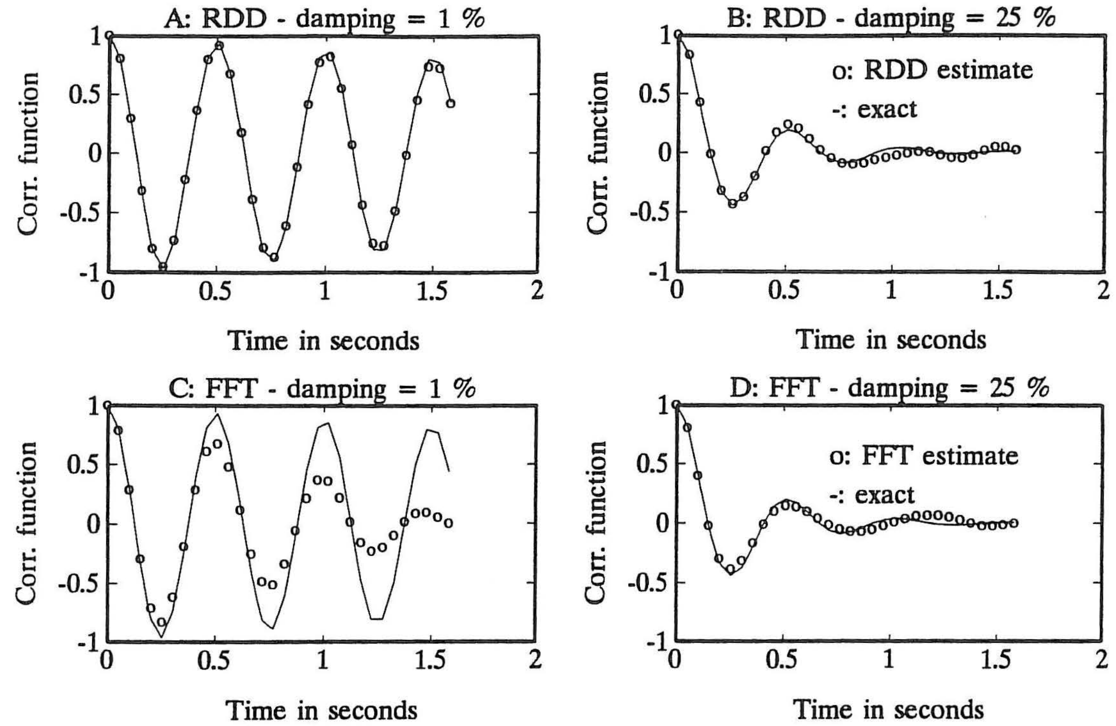

Figure 1. Autocorelation functions estimated by FFT and RDD technique for $M = 32$.

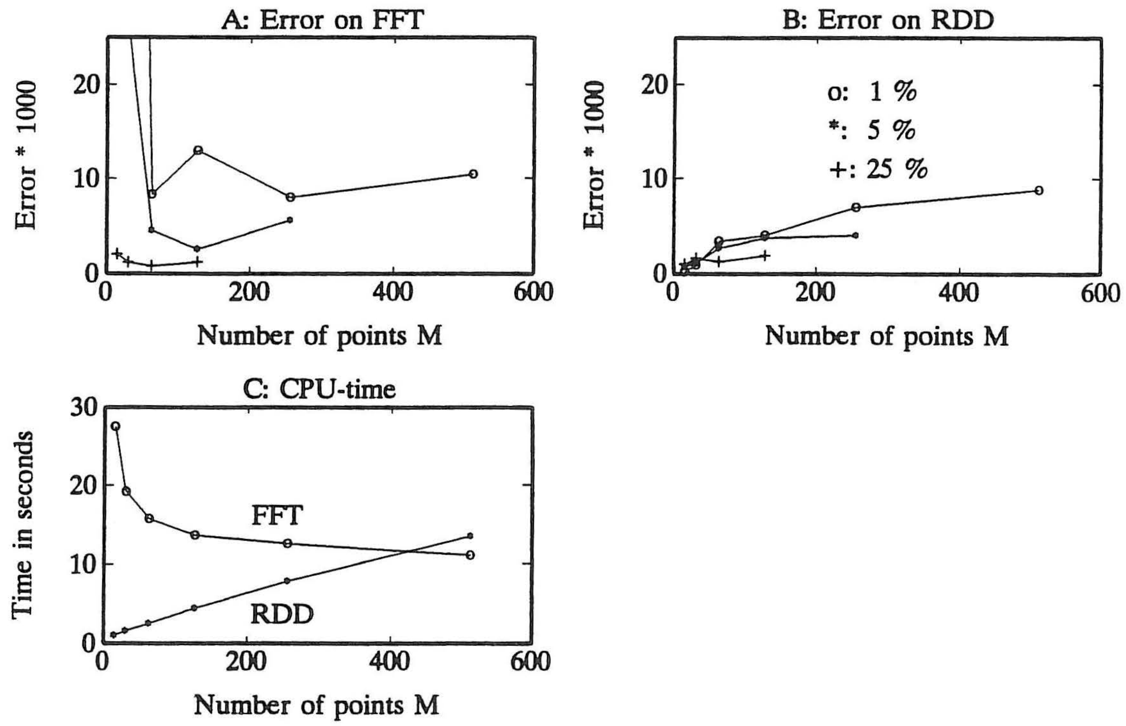

Figure 2. Errors and CPU-times when estimating autocorrelation functions by FFT and RDD.

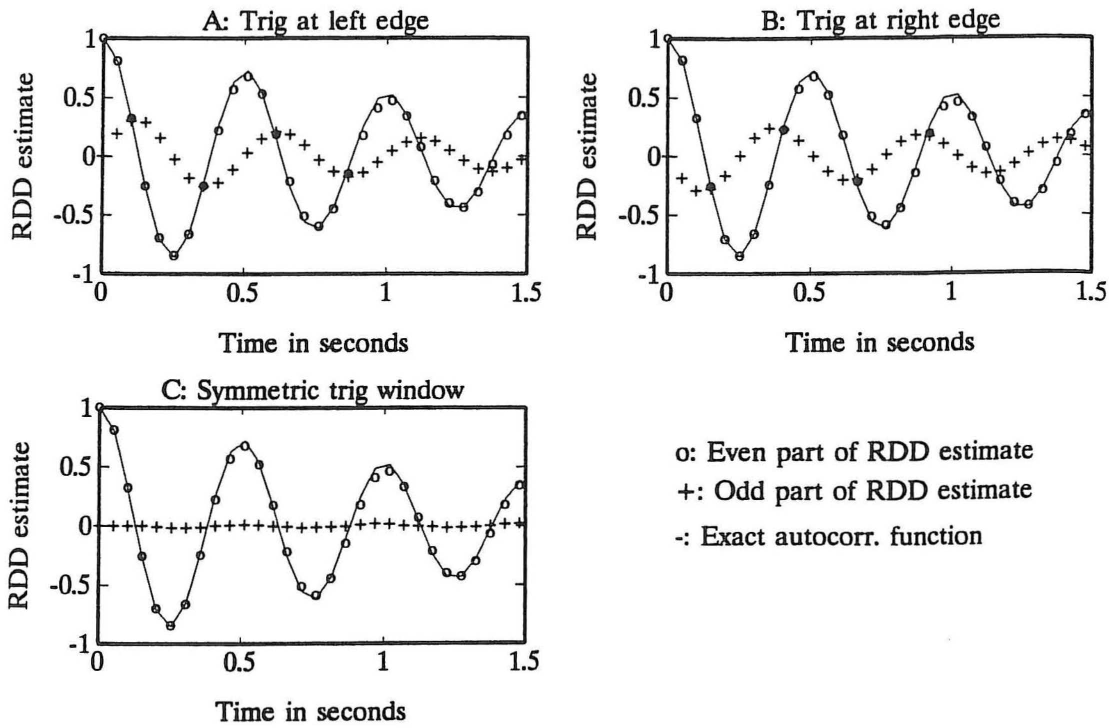

Figure 3. Bias introduced by the trig window.

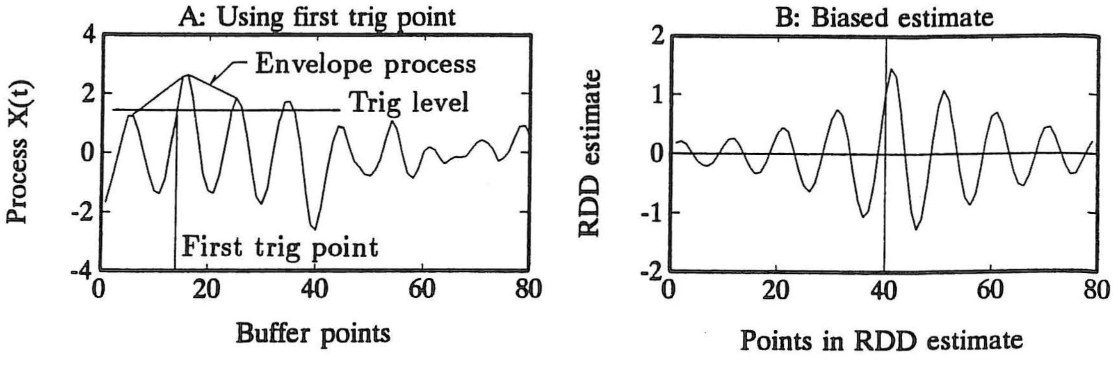

Figure 4. Bias introduced by selection of trig points.

explained in section 3. In figure 3 the even and the odd part of an RRD estimate of the auto covarianse function is shown, and as explained in section 3 the bias term (the odd part of the RRD estimate) is proportional to the derivative of the auto correlation function, figure 3.A and 3.B. However if both end points of the trig window are used as trig points, the resulting trig point falls in the midle of the window, and the bias disappears, figure 3.C

As mentioned in section 3 the exclusion of information by excluding trig points might allso introduce bias. This is illustrated by the following example. A time series of the usual length of 4000 points was divided into smaller series of 40 points each. Then the RDD estimate was found in the usual way (no window bias) except that trig points were searched in the small time series, and only the first trig point in each of the small time series was used. As it appears from the result shown in figure 4 the estimate is heavily biased.

In this case there are two resons for the bias. First, if the trig level is large (here equal to $1.5\sigma_X$) the first trig point will typically be an upcrossing (since the probability that $X > a$ at the first point in the time series is small). This will shift the estimate to the right along the time axis. Second, the first crossing of the envelope process will allso typically be an upcrossing (for the same reason), and therefore the system energy will be inreasing, causing the right part of the RDD estimate to get too large amplitudes (damping is underestimated) and the left part of the estimate to get too small amplitudes (the damping is overestimated). Both effects are significant in the example shown in figure 4.

In order avoid bias caused by exclusion of trig points, the time series should be large enough to hold a number of trig points that is large compared to the typical number of trig points between an upcrossing and a downcrossing of the envelope process, and all the detected trig points should be used.

5. Conclusions

The RDD technique is a versatile very simple non parametric technique for estimation of correlation functions. The theoretical basis of the technique has been established including fundamental solutions for Gaussian prosesses showing that RDD signatures allways will by a mixture of the corresponding correlation function and its derivative depending upon the the type of trig window used. Closed form solutions for the varianse of the estimate including contributions from finite size windows has been derived and bias problems has been illustrated.

Numerical investigations has shown that the techniue is fast and accurate compared to traditional FFT analysis especially for short estimates.

Acknowledgements

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APPENDIX A. Condition on Gaussian variables via regression

Let $\underline{X}$ and $\underline{Y}$ be Gaussian vectors, i.e. the elements $X_1, X_2, ...X_n; Y_1, Y_2, ...Y_m$ are jointly normal distributed and are therefore completely described by the expectation and the covariance

$$E\begin{bmatrix}\underline{X}\\ \underline{Y}\end{bmatrix} = \begin{bmatrix}\underline{\mu}_X\\ \underline{\mu}_Y\end{bmatrix} \quad ; \quad Cov[\begin{bmatrix}\underline{X}\\ \underline{Y}\end{bmatrix}[\underline{X}^T\ \underline{Y}^T]] = \begin{bmatrix}\underline{\underline{C}}_{XX} & \underline{\underline{C}}_{XY}\\ \underline{\underline{C}}_{YX} & \underline{\underline{C}}_{YY}\end{bmatrix} \tag{A.1}$$

We now define the vector

$$\underline{Z} = \underline{X} - \underline{\underline{B}}\,\underline{Y} \tag{A.2}$$

Since all the variables are Gaussian, $\underline{Z}$ will be independent of $\underline{Y}$ if and only if

$$\begin{aligned} Cov[\underline{Z}\ \underline{Y}^T] &= \underline{0} \\ &= \underline{\underline{C}}_{XY} - \underline{\underline{B}}\,\underline{\underline{C}}_{YY} \\ &\Downarrow \\ \underline{\underline{B}} &= \underline{\underline{C}}_{XY}\,\underline{\underline{C}}_{YY}^{-1} \end{aligned} \tag{A.3}$$

and we find the conditional expectation and covariance using that $\underline{Z}$ is independent of $\underline{Y}$

$$\begin{aligned} \underline{\mu}_{X|Y} &= E[\underline{X}|\ \underline{Y}] \\ &= E[\underline{Z} + \underline{\underline{B}}\,\underline{Y}|\ \underline{Y} = \underline{y}] \\ &= E[\underline{Z}] + \underline{\underline{B}}\,\underline{y} \\ &= \underline{\mu}_X + \underline{\underline{B}}(\underline{y} - \underline{\mu}_Y) \end{aligned} \tag{A.4}$$

$$\begin{aligned} \underline{\underline{C}}_{XX|Y} &= Cov[\underline{X}\ \underline{X}^T|\ Y] \\ &= Cov[(\underline{Z} + \underline{\underline{B}}\,\underline{Y})\ (\underline{Z} + \underline{\underline{B}}\,\underline{Y})^T|\ Y] \\ &= Cov[\underline{Z}\ \underline{Z}^T|\ Y] \\ &= Cov[\underline{Z}\ \underline{Z}^T] \\ &= \underline{\underline{C}}_{XX} - \underline{\underline{B}}\,\underline{\underline{C}}_{YX} \end{aligned} \tag{A.5}$$

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